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Forward and backward waves in the arterial system: impedance or wave intensity analysis?

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Abstract Both impedance analysis and wave intensity analysis are used to separate measured pressure and flow waveforms into their forward and backward components. The separation is sensitive to the characteristic impedance or wave speed determined from the data. In all other aspects, the results are identical.

1 Introduction

Impedance analysis and wave intensity analysis can appear to be antithetical theories for arterial haemodynamics. In fact, they are alternative modes of analysis of arterial pressure and flow based on the same basic physics. Neither is right nor wrong. Both are 1-D approximations that have different strengths and weaknesses. Ultimately, the choice of method depends upon the question that is being asked.

Quite naturally, the proponents of each method stress the advantages of their method and the weaknesses of the other. We propose to explore some of the common ground: the separation of the measured pressure and flow waveforms into their forward and backward components. We use the standard methods of separation on the same data and compare the results.

Both methods are dependent upon the determination of the wave speed c. In impedance analysis, this is generally done by determining the characteristic impedance Z_0 as the average of the impedance at higher frequencies where the higher dissipation means that reflected waves are dissipated before they return to the measurement site [7] or as the ratio of the change in pressure ΔP and the change in volume flow rate ΔQ during early systole, a time when wave reflections are assumed to be absent [3].

In wave intensity analysis, wave speed is used to separate waves. It is determined from the slope of the pressure-velocity (PU)-loop during the early part of systole, when wave reflections are assumed to be absent [4].

$$c = \frac{\Delta P}{\rho \Delta U}$$

Wave speed and characteristic impedance are linked by the water hammer equation

$$c = \frac{Z_0 A}{\rho}$$

Both methods can be seen to depend upon the determination of the wave speed. Traditionally, impedance analysis has been done using pressure and volume flow rate, whereas wave intensity analysis has been done using pressure and flow velocity. We will carry out our analysis in these traditional terms and note that their interconversion requires only knowledge of the area, since $Q = UA$, if viscous stresses are neglected.

2 Methods

2.1 Measurements

Pressure and flow velocity were measured simultaneously in the ascending aorta just before the subclavian bifurcation in a patient undergoing routine cardiac catheterisation for

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aortic insufficiency using a combined pressure/Doppler flow wire (Combowire, Volcano Therapeutics). The Doppler transducer is located at the tip of the catheter and the pressure transducer is 1 cm from the tip. The catheter was supported in the aorta by a 6F guide catheter and was manipulated to obtain the optimal velocity signal. Following acquisition, the pressure and velocity signal were aligned by shifting the velocity signal forward by 5 ms, as determined by in vitro calibration, to correct for phase lags due to the processing of the signals and the effect of the slight offset between the location of the two sensors. The diameter of the ascending aorta at end diastole was determined by echocardiography and the area A was calculated assuming that the vessel was circular. Data were recorded continuously for 60 s and ensemble averaged using the peak of the R-wave on the simultaneously recorded ECG as the fiducial marker. For the impedance analysis the measured U was converted to volume flow rate Q, but to allow direct comparison between wave intensity and impedance analysis in terms of flow separation, flow velocity rather than volume flow rate has been shown in the figures.

2.2 Data analysis

Separation of pressure P and volume flow rate Q waveforms into their forward and backward components was first proposed by Westerhof and his colleagues in 1972 [9]. Various algorithms for its implementation have been used since its introduction. We have used the algorithm proposed by Segers et al. [7]. Separation using wave intensity analysis was first proposed by Parker and Jones in 1990 [5] and their algorithm has been used.

2.2.1 Impedance analysis

The temporal waveforms are Fourier transformed and analysed in the frequency domain

$$\tilde{P}(f) = \mathcal{F}\{P(t)\} \quad \text{and } \tilde{Q}(f) = \mathcal{F}\{Q(t)\}$$

where t is time and f is frequency. The impedance is defined as the ratio

$$Z(f) = \frac{\tilde{P}(f)}{\tilde{Q}(f)}$$

The complex impedance is generally presented as the spectrum; the modulus and angle of Z as a function of f. The impedance measured at the inlet of an artery is termed the 'input impedance' and is complicated by the effects of reflection. The 'characteristic impedance' is defined as the impedance that would be observed in the absence of reflections. Z_0 was assessed in the frequency domain (average of harmonics 3–10 with exclusion of values >3 times the median value of Z over that range of harmonics). Because this method may introduce a bias to lower values of Z_0, we also calculated Z_0 in the time domain following an approach employed by Mitchell et al. [3].

Having determined Z_0, the method of separation of the waveforms is briefly

$$\tilde{P}_\pm = (\tilde{P} \pm Z_0\tilde{Q})/2$$

where $\tilde{P}_+$ represents the forward and $\tilde{P}_-$ the backward component of the Fourier transformed pressure. The components of the flow waveform are determined from the relationships

$$\tilde{Q}_\pm = \pm\frac{\tilde{P}_\pm}{Z_0}$$

The temporal separated waveforms are determined from the inverse transform of the respective frequency domain expressions

$$P_\pm(t) = \mathcal{F}^{-1}\{\tilde{P}_\pm(f)\} \quad \text{and } Q_\pm(t) = \mathcal{F}^{-1}\{\tilde{Q}_\pm(f)\}$$

2.2.2 Wave intensity analysis

The wave speed was determined from the measured P and U by plotting the PU-loop and determining the slope of the curve during early systole (over $\sim$40 ms following the foot of the pressure and flow upstrokes)

$$\rho c = \left(\frac{\mathrm{d}P}{\mathrm{d}U}\right)_{\text{early systole}}$$

Various methods have been described for determining the portion of the curve to analyse and its slope [1, 4]. In the following we have simply used visual inspection and manual measurement of the slope. The magnitude of successive waves dP and dU were taken as the differences between successive samples using a Savitsky-Golay filter to simultaneously smooth the data and determine the derivative [6].

Having determined ρc, the method of separation of the waveforms is briefly

$$\mathrm{d}P_\pm = (\mathrm{d}P \pm \rho c \mathrm{d}U)/2$$

where $\mathrm{d}P_+$ represents the forward and $\mathrm{d}P_-$ the backward component of the pressure. The components of the flow waveform are determined from the relationships

$$\mathrm{d}U_\pm = \pm\frac{\mathrm{d}P_\pm}{\rho c}$$

The separated waveforms are determined from the summation of the successive differences

$$P_\pm(t) = \sum \mathrm{d}P_\pm(t) \quad \text{and } U_\pm(t) = \sum \mathrm{d}U_\pm(t)$$

3 Results and discussion

The results of the analysis on data from the ensemble average beat are shown in Fig. 1. There is a slight difference between the results of the two separations, seen most easily in the difference in the magnitude of the flow. This is due almost entirely to the differences in the determination of the wave speed with the two different methods. The characteristic impedance determined by impedance analysis is $3,600\ \mathrm{kPas/m^3}$. The diameter of the aorta at the site of measurement was 3.3 cm. The wave speed determined from the characteristic impedance is therefore $c = 3.08$ m/s. The wave speed determined using wave intensity analysis is $c = 3.46$ m/s. In both cases we assume $\rho = 1{,}050\ \mathrm{kg/m^3}$.

As also shown by another paper in this issue [8], difference between the two analyses are minor. The differences arise primarily from the difference in the way that the wave speed is determined; the determination of the characteristic impedance, and the slope of the PU-loop. This difference is indicated in Fig. 2 which shows the data used to determine the wave speeds in the two methods.

Aside from the slight difference in magnitudes, the separation is essentially identical with both methods. This is not surprising given the mathematical similarity of the two methods, both being linearised 1-D models based upon the physical relationship between pressure and flow required to satisfy the conservation of mass and momentum in the absence of viscous stresses.

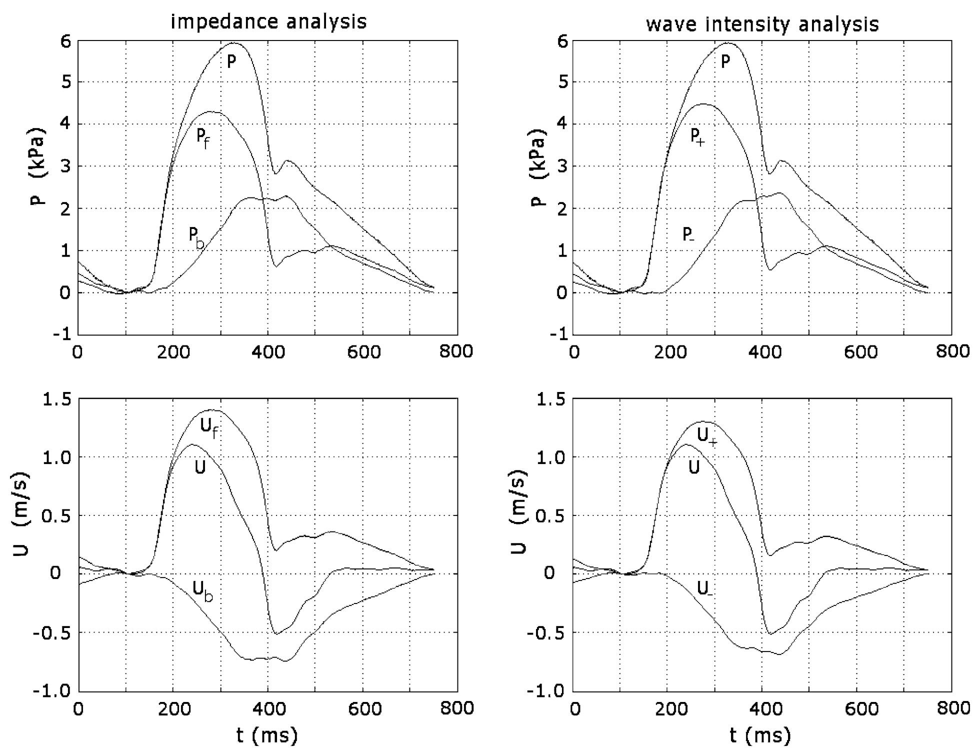

Fig. 1 The results of the impedance analysis (*left*) and wave intensity analysis (*right*) of the forward and backward waveforms. To allow for direct comparison of the separated flow, the impedance method results are presented as $U = Q/A$. Pressure is shown in the *top panels* and flow in the *bottom panels*. P measured P, P_f and P_+ forward P, P_b and P_- backward P, U measured U, U_f and U_+ forward U, U_b and U_- backward U

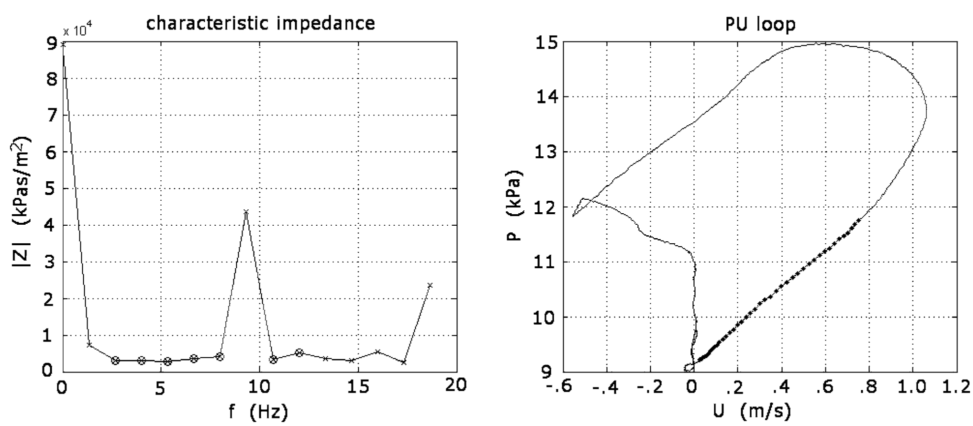

Fig. 2 The determination of Z_0 (*left*) and the determination of ρc from the PU-loop (*right*). Z_0 is defined as the average of Z over harmonics 3–10 excluding any value greater than 3 times the median (indicated by *circles*) [7]. The section of the PU-loop used to determine the slope is indicated by the *dots*

Finally, we note that local wave speed is determined by the physical characteristics of a blood vessel and inversely related to the local distensibility, $\mathcal{D} = \frac{1 \mathrm{d}A}{A \mathrm{d}P}$, [2]

$$c = \frac{1}{\sqrt{\rho \mathcal{D}}}$$

Since the density of blood varies only very slightly between individuals, there is a strong interrelationship between the characteristic impedance and the wave speed and the distensibility and compliance per unit length, $\mathcal{C} = \frac{\mathrm{d}A}{\mathrm{d}P}$, of the vessel. In terms of characteristic impedance

$$Z_0 = \frac{\rho c}{A} = \frac{1}{A}\sqrt{\frac{\rho}{\mathcal{D}}} = \sqrt{\frac{\rho}{A\mathcal{C}}}$$

or in terms of wave speed

$$c = \frac{AZ_0}{\rho} = \sqrt{\frac{1}{\rho \mathcal{D}}} = \sqrt{\frac{A}{\rho \mathcal{C}}}$$

These relationships, although straightforward, are not always appreciated.

We conclude that both impedance and wave intensity analysis give essentially identical results for the determination of the forward and backward waveforms. They are not interchangeable in other aspects of the respective analyses, one being done in the frequency domain and the other in the time domain. However, their correspondence in terms of wave separation indicates that they have more in common than might be apparent.

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