wave separation - 1

As we have seen, the wave intensity $dI = dPdU$ represents the net wave intensity due to both the forward and backward waves. If we now make the partially linearising assumption that the forward and backward waves are additive when they intersect, it is possible to extract even more information from the measurements.
If we define $dP_\pm$ and $dU_\pm$ as the changes in pressure and velocity in the forward '+' and backward '-' waves, additivity requires

$$dP = dP_+ + dP_- \qquad \text{and} \qquad dU = dU_+ + dU_-$$

These equations, together with the waterhammer equations for the forward and backward waves

$$dP_\pm = \pm\rho c dU_\pm$$

can be solved for changes in the forward and backward waves

$$dP_\pm = \tfrac{1}{2}(dP \pm \rho c dU)$$

or, equivalently,

$$dU_\pm = \tfrac{1}{2}\left(dU \pm \frac{dP}{\rho c}\right)$$

wave separation - 2

The forward and backward wave intensity for the separated waves is

$$dI_\pm \equiv dP_\pm dU_\pm = \frac{\pm 1}{4\rho c}\left(dP \pm \rho c dU\right)^2$$

It may not be immediately obvious, but a bit of simple algebra shows that the forward and backward wave intensity sum to the measured wave intensity

$$dI = dI_+ + dI_-$$

which is convenient.
The pressure and velocity waveforms for the forward and backward waves can be found by summing these wavefronts determined from the measured P and U

$$P_\pm(t) = \sum_0^t dP_\pm(t) + P_0 \qquad \text{and} \qquad U_\pm(t) = \sum_0^t dU_\pm(t) + U_0$$

where P_0 and U_0 are pressure and velocity at $t = 0$, effectively integration constants. This linearised form of separation of the waves into forward and backward components is formally identical to the Fourier method first proposed by Westerhof and his co-workers [Westerhof N, *et al. Cardiov. Res.* (1972) **6**, 648-656] and it produces essentially identical results.